

Pareto Explorer for Solving Real World Applications

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Abstract. An important characteristic of the Multi-objective Optimization Problems (MOPs) is that their solution sets typically form a $(k-1)$ -dimensional object where k is the number of objectives involved in the MOP. Thus, it is only possible to approximate the entire set of interest for a relatively few numbers of objectives (say, $k = 3$ or 4). In this work, we address the numerical treatment of MOPs with more than four objectives which are termed as Many Objective Optimization Problems (MaOPs). Such problems have recently caught the interest in the industry as the decision-making processes are getting more and more complex. The recently proposed Pareto Explorer (PE) method raises as a solution for the MaOPs, it is conceived as a global/local exploration tool which consists of two principal phases: obtaining a global optimal solution for a given MaOP, and the local exploration of optimal solutions based on the preferences of a decision-maker. In this work, we demonstrate the effectiveness of PE for solving real-world applications.

Keywords: many objective optimization, interactive method, decision making, continuation method.

1 Introduction

In many applications, several objectives have to be optimized concurrently leading to a *multi-objective optimization problem* (MOP). Due to the increasing complexity of practical problems, decision-making processes are getting more and more sophisticated. Motivated by the advances in the design of algorithms for the numerical treatment of MOPs [13] with few objectives and their huge success in applications, there is a recent trend to include more objectives into the optimization process. Due to this reason, MOPs with more than four objectives are often termed *many objective problems* (MaOPs) in the literature as they require a different numerical treatment than problems with two to four objectives.

However, there exist real-world problems where the decision-maker (DM) has some knowledge about the problem or she/he wants to obtain optimal solutions

with specific characteristics instead of a vast set of alternatives. Reference point methods are useful for this scenario, where the idea is to get the closest solution to a given vector, usually infeasible, which is a guess of the DM. This kind of methods, where the DM has active participation in the solution process, are called interactive methods, and they differ from each other according to what type of information they ask the DM [10].

We can find different alternatives which consider only one reference point to get a solution, some of them include reference point method [17], light beam search [8], GUESS [3], and even the r-NSGA-II [4], which can work with a set of points. On the other hand, the learning-oriented methods (a different class of interactive methods), exploit the preferences of the DM to direct the search, and reduce the number of solutions to consider. Such methods are useful when the set of optimal solutions is huge, for example, for many-objective optimization problems. A wide variety of these interactive methods have been developed [2], for example, Pareto Navigator [5], NIMBUS [12], and Nautilus [11].

Recently, continuation methods have been used to solve the multiobjective optimization problem. These methods have the advantage that they move through the Pareto front. To achieve this, we need an initial optimal solution, starting from this point we compute a predictor, which is a movement according to specific criteria, and then with a corrector we obtain a new optimal solution. The change both in the predictor as in the corrector, gives rise to different methods as they are Hillermeier method [7], Pareto Tracer [9], and Zigzag [16].

The method used this paper, called Pareto Explorer, is a continuation method that was recently proposed and which takes into account the preferences of the DM to calculate the predictor. It is in spirit an interactive method and even more. Here we show how to use PE to solve real world applications.

2 Background

A continuous multi-objective optimization problems (MOP) is mathematically expressed as:

$$\min_{x \in D} F(x) = [f_1(x), \dots, f_k(x)]^T, \quad (1)$$

where $D \subset \mathbb{R}^n$ is the domain and $F : D \subset \mathbb{R}^n \rightarrow \mathbb{R}^k$ is called the objective function, where k is the number of objectives and n is the number of variables.

The optimality of an MOP is defined by the concept of *strict dominance*. Let $v, w \in \mathbb{R}^k$, the vector v is *less than* w ($v <_p w$), if $v_i < w_i$ for all $i \in \{1, \dots, k\}$; the relation \leq_p is defined analogously. A vector $y \in D$ is *dominated* by a vector $x \in D$ ($x \prec y$) with respect to (1) if $F(x) \leq_p F(y)$ and $F(x) \neq F(y)$, else y is called non-dominated by x . A point $x^* \in \mathbb{R}^n$ is Pareto optimal to (1) if there is no $y \in D$ that dominates x . The set of all the Pareto optimal points P_D is called the Pareto set and its image $F(P_D)$ is called the Pareto front. Typically, i.e., under certain mild smoothness assumption on the model, both Pareto set and front form at least locally $(k - 1)$ -dimensional objects.

2.1 Pareto Tracer

Here we briefly state the core elements of PT for unconstrained problems, for details including constraint handling we refer to [9].

In [7] we find a continuation method for the MOPs context by considering $\hat{F} : \mathbb{R}^{n+k} \rightarrow \mathbb{R}^{n+1}$:

$$\hat{F}(x, \alpha) = \begin{pmatrix} \sum_{i=1}^k \alpha_i \nabla f_i(x) \\ \sum_{i=1}^k \alpha_i - 1 \end{pmatrix} = 0. \quad (2)$$

The set of KKT points of (1) is contained in the zero set of \hat{F} which motivates the continuation along $\hat{F}^{-1}(0)$.

The idea of Pareto Tracer [9] is to separate the decision and weight space:

$$\hat{F}'(x, \alpha) \begin{pmatrix} \nu \\ \mu \end{pmatrix} = \begin{pmatrix} \sum_{i=1}^k \alpha_i \nabla^2 f_i(x) & \nabla f_1(x) & \dots & \nabla f_k(x) \\ 0 & 1 & \dots & 1 \end{pmatrix} \begin{pmatrix} \nu \\ \mu \end{pmatrix}. \quad (3)$$

By the second equation of (3) we have that $\sum_{i=1}^k \mu_i = 0$, and it is possible to find a relationship between ν and μ , i.e., a relationship between the objective space and the variable space:

$$\nu_\mu = -W_\alpha^{-1} J^T \mu, \quad (4)$$

where $W_\alpha := \sum_{i=1}^k \alpha_i \nabla^2 f_i(x) \in \mathbb{R}^n$ and $J = J(x) = (\nabla f_1(x)^T, \dots, \nabla f_k(x)^T)^T \in \mathbb{R}^{k \times n}$. Finally, given a direction $d \in \mathbb{R}^k$ in objective space such that $J\nu_\mu = d$, this vector ν_μ can be obtained with the vector μ_d that solves:

$$\begin{pmatrix} -JW_\alpha^{-1} J^T \\ 1 \dots 1 \end{pmatrix} \mu_d = \begin{pmatrix} d \\ 0 \end{pmatrix}. \quad (5)$$

If the rank of J is $k - 1$, we can compute the set of tangent vectors via a QR factorization of α , i.e. $\alpha = QR$. Let Q_2 denote the matrix formed by the last $k - 1$ columns vectors of Q , this matrix is an orthonormal basis of the linearized Pareto front at $F(x)$.

3 Pareto Explorer

In case the number k of objective is too high, it is not possible to compute a suitable finite size approximation of the entire solution set any more. Instead, the Pareto Explorer [14] aims to find a solution in cooperation with the DM in two steps:

Step 1 Compute a solution x_0 of the MaOP.

Step 2 Explore the Pareto landscape around x_0 via performing movements into user specified directions.

Step 1 can be performed via a global heuristic such as an evolutionary reference point method. For Step 2, the above described PT has been adapted in [14] that allows to perform best fit movement along the Pareto set/front in directions defined in decision, objective, and weight space. The key for this is the fact that the tangent spaces of both the Pareto set at x as well as the Pareto front at $F(x)$ can be computed for every regular solution x which follows by the above discussion (computation of the predictor for PT). By doing so, Step 2 allows for a fine-tuning of the initial solution x_0 from Step 1.

Let $d_k \in \mathbb{R}^k$ be a given direction, if x is a solution of (1) with its corresponding α vector, then the best direction to move the point $F(x)$ on the Pareto front, according d_k , is given by the orthogonal projection of d_k on the linearization of the Pareto front at the point $F(x)$ i.e:

$$d = Q_2 Q_2^T d_k, \tag{6}$$

where Q_2 are the last $k-1$ columns of the QR factorization of α . We consider the normalization of d , i.e. $d = d/\|d\|$ and we can now compute the desired vector $\nu_d \in \mathbb{R}^n$ such that $J\nu_d = d$ using (4) and (5).

The normalization of d is useful in order to compute the step length t for the predictor, if we want that, for two consecutive solutions, $\|F(x_i) - F(x_{i+1})\| \approx \tau$, then t is given by:

$$t = \frac{\tau}{\|J\nu_d\|}. \tag{7}$$

The corrector is, as in the case of the Pareto Tracer, given by the Newton method for MOPs [6].

Figure 1 shows a hypothetical example for a best fit movement along the Pareto front from the image $F(x_i)$ at the current iterate x_i . Hereby, $d_y \in \mathbb{R}^k$ denotes the desired direction in objective space specified by the DM, and $d_y^{(i)} \in \mathbb{R}^k$ the direction projected to the linearized Pareto front at $F(x_i)$. The projected direction $d_y^{(i)}$ is used to perform a best fit movement along the Pareto front of the problem.

4 Applications

In this section, we illustrate the efficiency of our method via two real world applications, the industrial laundering and the plastic injection molding (PIM).

4.1 Industrial Laundering

The laundering process is influenced by the four parameters temperature, chemistry (amount of cleaner), time and mechanics (speed of rotation), which is described by Sinners' Circle [15]. These laundries are capable of washing up to 30 tons of laundry per day. Consequently, it is of great interest to increase the efficiency, which is beneficial both for ecological as well as economic reasons.

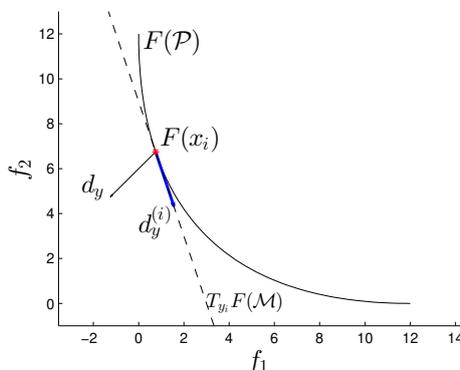


Fig. 1. Best fit direction $d_y^{(i)}$ for a given direction d_y in objective space for the Pareto Explorer.

The model was generated by fitting quadratic Ansatz functions to measurements. The decision variables are the temperature of the water (x_1), the amount of washing detergents (x_2), the washing time (x_3) and the rotating speed of the laundry (x_4). Thirteen of the objectives are related to the cleaning of specific types of contamination, i.e., f_1 to f_{13} represent the effectiveness on: wool grease in cotton, wool grease in polyester, red in cotton, sebum in cotton, sebum in polyester, curry in cotton, motor oil in cotton, petroleum in cotton, blood in cotton, egg in cotton, starch in cotton, cocoa, and vegetable grease, respectively; while the 14th objective is related to the negative of the cost. All parameters are normalized with the reference point being at $(0, 0, 0, 0)$. The degree of cleaning varies between 0 (no cleaning) and 100 (perfect cleaning). This leads the following model:

$$\begin{aligned} \min_{x \in \mathbb{R}^4} F(x) &= [f_1(x), \dots, f_{14}(x)]^T, \\ \text{s.t } -1.5 &\leq x_i \leq 1.5, \quad i = 1, 2, 4 \\ &0 \leq x_3 \leq 1.5. \end{aligned} \tag{8}$$

We used Pareto Explorer to solve the problem of the washing machine (for more details see [14]). For this approach we define the direction in objective space as $d_k = -e_{14}$, i.e. we want to reduce as much as possible the value of the 14th function, which is the cost function. We took as initial point the optimal vector $x_0 = (1.0429, 0.8521, 1.3622, 1.5000)$ and $f_{14}(x_0) = 9.5637$. We obtained such a point, after applying the Newton method at the vector formed by the middle value for each variable.

In order to view more easily the obtained results, we introduce graphs which represent optimal solutions as polygons inscribed in the unit circle. In all cases, the center of the circle depicts the best values for each objective function. Then, the more a solution is far to the center, the best is its value. The first objective function is in the line which goes from the point $(0, 0)$ to the point $(1, 0)$, the rest of the objective functions are set in the counter-clockwise direction.

The result obtained by the Pareto Explorer is in Figure 2a (left), the method did 110 iterations and we show the solution for the initial point, 27th iteration, 55th iteration, and the final result. We can see that there is a reduction of the value for the function f_{14} until reach the best value. The values of f_{14} and the points considered in Figure 2a are:

- $x_0 = (1.0429, 0.8521, 1.3622, 1.5000)$, $f_{14}(x_0) = 9.5637$,
- $x_{27} = (1.0995, -0.5341, 1.5000, 1.5000)$, $f_{14}(x_{27}) = -4.2412$,
- $x_{55} = (0.7417, -1.2346, 1.5000, 1.5000)$, $f_{14}(x_{55}) = -11.6041$,
- $x_{110} = (-1.4718, -1.5000, 1.5000, 1.5000)$, $f_{14}(x_{110}) = -16.5000$.

4.2 Plastic Injection Molding

The parameters we consider are the melt temperature (T_{melt}), the packing time (t_{pack}), the packing pressure (P_{pack}) and the cooling time (t_{cool}). While the seven objectives are related to the quality and productivity of the PIM process. Cosmetic characteristics are measured by the *warpage* (f_1) in the product, *shrinkage* (f_2) and *sink marks* (f_3). Functional properties are represented by residual stresses such as *Von Mises* (f_4) and *shear stresses* (f_5). Productivity is measured by the *cycle time* (f_6) and *clamping force* (f_7). Commonly, only between two and four of these objectives are considered in other works (for more details see [1]).

As case study we use in this work the design of a particular plastic gear. The model (obtained by a surrogate model) is the following:

$$\begin{aligned} \min_{x \in \mathbb{R}^4} F(x) &= [f_1(x), \dots, f_7(x)]^T, \\ \text{s.t } 190 &\leq x_1 \leq 230, \\ 3 &\leq x_2 \leq 5, \\ 60 &\leq x_3 \leq 100, \\ 8 &\leq x_4 \leq 14. \end{aligned} \tag{9}$$

Here we consider the seven described objectives and as initial solution we chose $x_0 = (210.00, 4.00, 80.00, 11.00)^T$. Again, it is the middle point for each variable in the considered range. For the demonstration of Step 2 of the PE, we use the scenario in which we want to minimize the functions f_1 , f_5 , and f_6 at the same time, i.e., the direction is $d_y = (-1, 0, 0, 0, -1, -1, 0)^T$ with $\tau = 0.01$.

We can see in Figure 2b that the functions f_1 and f_6 are directly in conflict, while f_5 the value depends of both functions. At the end of the optimization process, we obtain the best value for f_6 and the worst value for f_1 ; for the case of f_5 the initial and the final values are similar, but along the steps such value has a lot of variation. Notice that, the result for this scenario is almost the same than the previous one.

As it can be seen, the movement has been performed according to the desired direction. We have presented here the entire path of solutions, however, in a real decision-making process, the DM can of course chose at any time either to accept a computed candidate solution, or to change the direction in which the steering has to be performed.

Table 1. Model values (F_M) against the simulated values F_S for the PIM.

Initial Configuration							
x_0	210.0000	4.0000	80.0000	11.0000			
$F_S(x_0)$	0.2016	5.6565	9.7470	0.0717	0.8690	20.1000	11.9460
$F_M(x_0)$	0.2040	5.7271	9.7329	0.0713	0.8774	20.1000	11.8221
Final Configuration							
x_{212}	213.3452	3.3421	60.0000	9.6950			
$F_S(x_{212})$	0.2437	6.5210	9.1729	0.0772	1.0300	18.1371	7.9492
$F_M(x_{212})$	0.2419	6.4289	9.6057	0.0770	0.9199	18.1371	11.7847

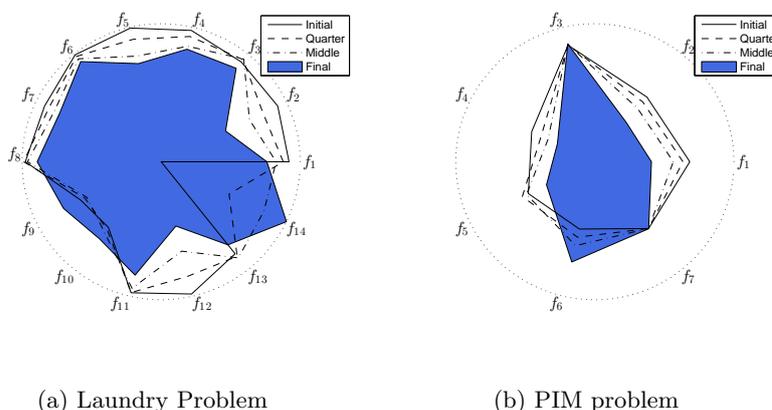


Fig. 2. Graphical results.

5 Conclusions and Future Work

In this paper, we present an overview of how to use the Pareto Explorer, a global/local exploration tool for the effective numerical treatment of many objective optimization problems, to solve real world applications in the context of the decision-making process. We use it because it is not possible to compute a suitable finite size approximation of the *entire* Pareto set/front for problems with many objectives. Instead, solutions are computed and presented to the DM in a two stage approach, where he/she express the preferences as a direction in objective space. We demonstrated the effectiveness and usefulness of this method with two real-world applications. The use of applications is essential in the context of interactive methods because making fair comparisons is not always possible, due to the fact that each process requires different pieces of information. Moreover, comparisons of the PE against other continuation methods is unfair, as they try to approximate all the set of optimal solutions.

However, the applicability of PE is restricted to continuous MaOPs. As future work, the adaptation of PE for problems with different smoothness assumptions can be explored.

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