

Signal Timing Model for Traffic Intersection Control

Diego Uribe¹, Enrique Cuan¹, Salvador Ibarra², Javier Diaz³

¹ Instituto Tecnológico de la Laguna,
División de Estudios de Posgrado e Investigación, Torreón, Coah.,
Mexico

² Universidad Autónoma de Tamaulipas,
Mexico

³ Instituto Tecnológico de Celaya,
Mexico

diego@itlalaguna.edu.mx, kcuand@itlalaguna.edu.mx,
sibarram@docentes.uat.edu.mx, javier.diaz@itcelaya.edu.mx

Abstract. In this article we analyze the basic elements to be considered for the control of a traffic light by using modelling based on agents. More specifically, we model the factors that affect the way traffic engineers determine an appropriate sequence of signal indications at a busy intersection. In this model the user controls the flow of vehicles coming from each direction, the speed of the cars and the cycle-length at the traffic light. By trying different values of traffic flow while keeping the cycle-length fixed, the user can analyze the impact of the traffic flow in each direction under a particular cycle-length at the traffic signal. Also, this work makes clear how the use of agent-based modeling to simulate the control of a traffic light proves to be a useful tool for the analysis previous to the final installation of a traffic signal.

Keywords: Agent-based modeling, crossroad, intersection, traffic light, cycle-length, phase.

1 Introduction

It was in 1914 when the installation of the first traffic light in the city of Cleveland brought a significant improvement in the control of a transportation system. This visual signal device created for orderly managing the movement of vehicles was a watershed in the transportation industry: it offered us efficient and safe transportation. But nowadays, the vehicular explosion in big cities is the cause of many urban problems: wasted time as a consequence of traffic-related delays, poor fuel efficiency, high pollution, and unfortunately, an increase of people under stress [5]. So the need for a network of intelligent transportation system to be able to cope with these multiple problems is evident.

One of the most important activities of a traffic engineer is to measure the concentration of vehicles at points where two or more roads meet in order to avoid vehicle conflicts as well as to optimize the vehicle flow. Among the multiple strategies for handling traffic intersections, traffic light is the most common alternative to regulate vehicular traffic in all major cities around the world. One of the reasons why traffic signals are so common is because they can be programmed for the orderly movement of vehicles and pedestrians. In other words, for a traffic engineer the control of the traffic signal is responsive to the demands of the traffic flow.

A vehicular traffic simulator plays a central role in the toolkit of a traffic engineer: it is an essential tool because it allows analysing hypothetical scenarios corresponding to several possible vehicular flows and the estimation of the cycle-length at the traffic light. In fact, by using a traffic simulator, a traffic engineer embarks on the pre-analysis of an urban area to be investigated in order to make decisions such as expanding the number of lanes, installing traffic lights, or altering the scheduled times of existing traffic lights. Thus, the purpose of using a traffic simulator is to establish various hypotheses to be verified, or disproved, based on the road traffic information obtained in real time.

According to the definition of a complex system as a system characterized by an environment in which multiple individual and independent elements interact with each other giving rise to an emerging phenomenon [6,8], to analyze vehicular traffic from this perspective is rather a plausible approach. In fact, taking into account the multiple factors to be considered when driving a vehicle (e.g. the presence of other vehicles, road conditions, traffic lights, traffic accidents, pedestrian crossing), a traffic system may also be characterized as a complex system.

In this way, since agent based modeling is a computational methodology that allows us to model complex systems [18], we analyze in this work the basic elements to be considered in vehicular traffic by using modeling based on agents. To be more specific, the vehicles and the traffic lights in our model are represented as agents, whereas the roads of the intersection are represented as the environment in which the vehicles travel and interact. Also, a set of basic parameters is defined to regulate the behavior of the agents and to observe the impact of different values on the interaction of multiple distributed elements.

For example, in this model the user controls the flow of vehicles coming from each direction, the speed of the cars and the cycle-length at the traffic light. By trying different values of traffic flow while keeping the cycle-length fixed, the user can analyze the impact of the traffic flow in each direction under a particular cycle-length at the traffic signal. Last but not least, this work makes clear how the use of agent-based modeling to simulate the control of a traffic light proves to be a useful tool for the analysis previous to the final installation of a traffic signal.

The theoretical framework of the methodology adopted in the traffic model is described in section 3. As it is briefly mentioned lines above, the model represents the simulation of a traffic intersection control based on agents so the properties,

behavior and interactions between autos and traffic signals are detailed. Then, the section 4 describes the impact of each parameter on the waiting time of cars through the intersection by adjusting some values while keeping fixed others when we run the simulator. But first we briefly review in the next section other works that have been developed for the control of a traffic intersection.

2 Related Work

In this section, we briefly narrate other models for describing traffic flows. Traffic flow theory is a scientific field relatively young. The first traffic problems in the past century were solved by the implementation of rule of thumb methods. However, at the beginning of the 1950s, the turning point in the field of traffic engineering came when Glen Wardrop described traffic flows by using mathematical representations [14].

The first popular traffic flow model, commonly known as the LWR model, was based on fluid dynamics, a discipline for studying fluids (liquids and gases) in motion [9,12]. In fact, traffic flow model makes use of numerical analysis and algorithms to solve and analyze problems that involved fluid flows. The interactions of liquids and gases are used as an analogy to model the interactions among vehicles. Another interesting work was the traffic model implemented by Prigogine and Herman who drew an analogy between gas-kinetic and the velocity distributions [11]. It is basically a stochastic model of traffic flow in which the “microgoals” of each driver are modified by the interaction with other drivers. Then, the central idea is to predict how much the “microgoals” of the driver have been modified by making use of a kinetic equation defined in terms of the desire of the driver and the interactions with other vehicles.

Among a myriad of interesting works about traffic control, Gershenson proposes the use of self-organizing methods as strategy for traffic signal control [4]. Since traffic densities change constantly, this research is based on the premise that self-organizing methods offer an adaptive alternative to traditional and rigid control methods. Traffic lights respond to car densities by implementing rules to give priority to cars waiting for a long period. As a self-organizing system is described as one in which elements interact in order to achieve a global function, in this work we can see how cars and traffic lights interact to share the control of the signals for improving traffic flow.

Another interesting adaptive intersection control system is based on cooperative conflict resolution techniques. Specifically, Ball and Dulay proposed a distributed approach using vehicles as intelligent objects to encourage interaction to safely and efficiently regulate intersections [1]. By using vehicles capable of both sensing and actuating within their local space, they are able to communicate with each other using wireless technology. Thus, vehicles cooperate using ad-hoc messaging to share journey plans to safely travel over a shared intersection. In this way, the intersection control system contributes to a plausible reduction in the number of start-stops and the total travel distance.

3 Theoretical Framework

In this section we describe the theoretical framework that supports the basic signal timing model to be analyzed in this article. First, we define a set of essential terms to be considered in the design of a proper timing of the signals for the efficient operation of the traffic light. Then, we briefly present the staple concepts of agent-based modelling, the methodology for designing and building the signal timing model.

3.1 Signal Timing Concepts

Intersections, places where two or more roads join or cross each other, can be divided in terms of their form and their handling or operation. Since the control or operation of the intersection is the research topic of this paper, different strategies for the control of conflicting traffic movements are available to the engineer: priority control, space-sharing, time-sharing and grade-separated [10].

Priority intersections denote a crossroad where the minor road traffic enters the main road stream during spare time gaps. The access to the main road is normally controlled by stop or give way signs and markings on the minor road. The principal advantage of this type of intersection is that the stream of vehicles on the main road are not delayed.

Space-sharing intersections are commonly known as roundabout intersections. These particular type of intersections denote a place where three or more roads join and traffic must go around a circular area in the middle, rather than straight across. The access to the intersection is normally controlled by give way markings since vehicles circulating in the roundabout have priority.

Grade-separated intersections are also known as interchange intersections. These particular type of intersections denote a system of connector roads for the interchange of vehicular traffic between two or more roads on different levels. The access to the main stream is through the use of connection roads known as slip roads. The principal advantage of this type of intersection is that minimizes the number of conflict points.

Time sharing intersections represent the type of intersection to analyze in this paper. These particular type of intersections operate on the basis that separate time periods are allotted to conflicting traffic streams so that each can make safe and efficient use of the intersection at different times. Since the operation of this intersection relies on separate time periods for the orderly movements of vehicles, the definition of a proper timing of the traffic light signals is imperative. In fact, the definition of the intervals of a traffic signal is based on a set of essential terms to be described next [2,13]:

- *Cycle*: it is defined as a complete color sequence of signal indications. For example, the sequence green-yellow-red denotes a cycle.
- *Cycle length*: the time required to complete a cycle is defined as cycle length. For example, the cycle length is the time that elapses from the start of the green indication to the end of the red indication.

- *Phase*: set of traffic streams having the right-of-way simultaneously before the release of another conflicting set of movements at an intersection.
- *Interval*: a period of time during which no signal indication changes.

The cycle has four main components:

- *Green interval*: a period of time during which a set of traffic streams can effectively move. All other movements have red indication.
- *Yellow interval*: a period of time required for a driver traveling on a particular approach to realize the permission for movement is about to end so the driver can safely stop or safely go through the intersection.
- *Clearance interval*: this is a red indication provided after the yellow interval to give additional time to those vehicles that enter the intersection on yellow before conflicting flows are released. It is also called the "all red interval."
- *Red interval*: a period of time during which a set of traffic streams are not allowed to move while other conflicting flow streams are allowed.

For the sake of illustration of the above terms and concepts, Figure 1 shows a simple four-arm intersection where two approaches meet: East/West and North/South. Figure 1 also shows traffic movements in a two phase signal system with straight traffic without left turning movements. Since East and West streams are non-conflicting movements, they are grouped in a single phase, whereas the non-conflicting North and South streams are grouped in a second phase.

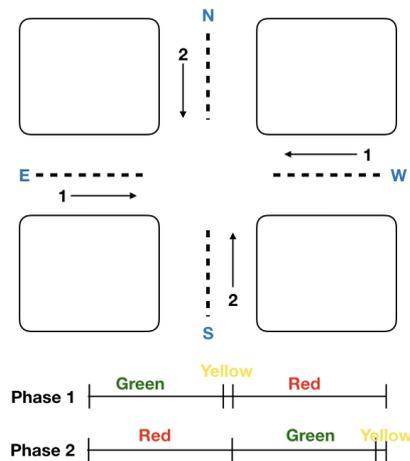


Fig. 1. Two-phase signal system.

Once the number of phases in the intersection have been defined, as well as the yellow and clearance interval together with the cycle length, an adaptable signal timing light can be programmed. First, the critical volume for each each

phase and the total critical volume are determined. The critical volume is the maximum of the flows corresponding to each phase (cvp), whereas the total critical volume (cvt) is just the sum of the critical volumes.

$$cvt = \sum_{i=1}^n (cvp_i), \quad (1)$$

where n denotes the number of phases and

$$cvp_i = \max (f_i), \quad (2)$$

where f_i denotes a particular flow or volume for p phase.

Second, the green time for each phase is calculated. Basically, the green interval represents the proportion of the cycle length to be allocated to each phase. Thus, the length of the green period for each phase (gp_i) is obtained from:

$$gp_i = \frac{cvp_i}{cvt} \times cl, \quad (3)$$

where cl denotes the cycle length.

Finally, the red interval for each phase is calculated in terms of the yellow and clearance interval as well as the green interval corresponding to the rest of the phases.

3.2 Agent-based Modeling

Agent based modeling is a computational methodology that allows us to model complex systems [18]. A complex system is defined as a system characterized by an environment in which multiple individual and independent elements interact with each other giving rise to an emerging phenomenon [6] [8]. ABM is a computational modeling paradigm that enable us to describe a complex system in terms of agents, environment, and interactions. While agents denote the basic ontological unit of the model, the environment represents the world in which the agent lives. In this work, we make use of agent based modeling to represent the interaction of vehicular flows concurring in a traffic intersection. To be more specific, vehicles and traffic signals are represented as agents, whereas the junction of the streets is represented as the environment in which the vehicles travel and interact. Simple functions such as acceleration and breaking characterize the vehicles behavior, whereas to change the signals according to a fixed or variable strategy characterises the behavior of a traffic light.

4 Signal Timing Simulations and Discussion

Once the basic concepts to be considered in the design of a proper signal control have been described, we present the experimentation implemented with two multi-agent models that represent the two principal types of signal control: pre-timed and actuated. As these agent-based models enable us to analyze

the range of behaviors exhibited by multiple simulations, we first present the results obtained from fixed signal timing simulations, and then, the results obtained from adaptable signal timing simulations. In both cases, we describe the parameters of the model, as they are used as the primary tuning knobs to determine particular settings for specific vehicular flows concurring in the intersection.

Now, in order to analyze the simulation results, we need to measure how effective the settings of traffic signals are. As one of the primary functions of traffic control signals is to reduce the vehicular wait for the right-of-way, delay is the most common measure of effectiveness for signalized intersections [15]. Thus, when we run one of the models, the computer simulation provides the overall number of stopped cars. By counting the number of vehicles in the queue at fixed intervals of time, the traffic engineer is able to measure the effectiveness of a particular configuration of the model's parameters. We now proceed to the description of the results obtained from fixed signal timing simulations.

The intersection control models used to illustrate the types of traffic signal control have been developed with NetLogo [17]. NetLogo is a programming language used to create models based on agents and has also proved to be a well suited tool for modeling complex systems evolving over time. One of the most appealing features of NetLogo is the capacity to investigate the connections between the micro-level behavior of individuals and macro-level patterns that emerge from their interactions [7].

4.1 Fixed Signal Timing Simulations

In this case, we extended the traffic intersection model presented by Wilensky [16]. Such model represents a 2-phase independent intersection with a pre-timed control characterized by a repetitive cycle and split timing. Figure 2 shows the context in which, on the left side, three purple buttons allow to establish the scenario (setup) and execute the model (go and go once). Likewise, a set of green sliders allow the configuration of the traffic conditions to be simulated. The first three sliders concern the control of the speed of the vehicles (speed-limit, max-accel and max-brake), the next two sliders determine the traffic volume of each approach (flow-North and flow-East), and the last two sliders allow to set up the time in seconds corresponding to the green and yellow interval respectively.

The results shown in Table 1 were obtained for a particular green interval (16) and three different values of traffic volume distribution (60, 80 and 100) for the North approach, whereas the flow for the East approach was fixed at 100. Initially, when we have a traffic volume of 60 at the North approach, we are simulating an intersection where the North approach represents a minor road and the East approach denotes the major street. When we increase the traffic volume at the North approach with a value of 80 we still have an intersection with a minor and major street. However, when we finally increase the traffic volume at the North approach with a value of 100, we are simulating an intersection of streets with equal traffic volumes.

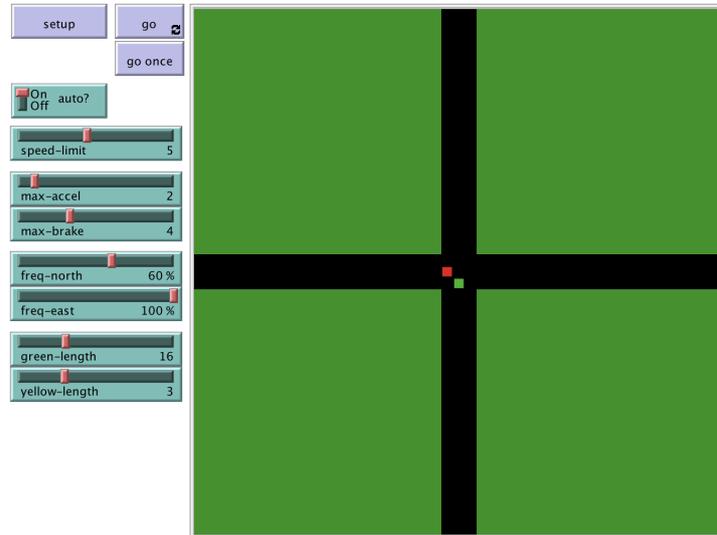


Fig. 2. Intersection-green-2-phases.

Another important point to be noticed in the Table 1 is the column labeled as Ticks. This column represents the time in seconds to run each simulation and the values in parenthesis denotes the number of cycle length to be observed. For example, 600 corresponds to 15 times 40 where 15 denotes the number of signal cycles and 40 represents the value of the cycle length when the green interval is 16, yellow interval is 3 and clearance interval is 1. Thus, we run simulations under complete rotations of the signals for each approach so the same number of color signals was allotted to each street. The upshot of this small but significant extension of the model has been to be able to more precisely measure the number of stopped cars and therefore, to understand the circumstances under which a fixed signal control is appropriated.

We now see how, as long as the traffic volume on the North approach (values of 60 and 80) is less than the traffic volume on the East approach (values of 100), the number of stopped cars on the major street (East road) is higher than on the minor street (North road). As a consequence of this traffic distribution, traffic on the major street forms into platoons of slow-moving vehicles which may stop and start (denoted by values greater than one). Thus, the green interval for the traffic volume on the major street is not enough. On the other hand, as the concentration of traffic on the North approach is low, drivers on the minor road are not disturbed by additional delays at the traffic light (denoted by values lower than one). In other words, the green interval for the traffic volume on the minor street is sufficient.

Now, when we finally increase the traffic volume at the North approach with a value of 100, we are simulating an intersection of streets with equal traffic volumes. Since the traffic distribution at both overlapping approaches is high

Table 1. Signal-fixed 16.

Traffic Distribution	Ticks	Flow of cars		Stopped cars			
		East	North	East	North	%East	%North
60 North - 100 East	600 (15)	229	136	414	42	1.81	0.31
	1200 (30)	443	264	840	86	1.90	0.33
	2400 (60)	862	516	1696	162	1.97	0.31
80 North - 100 East	600 (15)	215	171	440	70	2.05	0.41
	1200 (30)	420	336	869	99	2.07	0.29
	2400 (60)	841	672	1718	206	2.04	0.31
100 North - 100 East	600 (15)	223	231	419	413	1.88	1.79
	1200 (30)	423	443	872	849	2.06	1.92
	2400 (60)	848	863	1718	1695	2.03	1.96

and alike, the number of stopped cars is also similar. By running the model we can see how platoons of slow-moving vehicles are formed at both streets and also disturbed by delays. As the simulation results show, the green interval for high traffic volume on both streets is not enough: a substantial increase on the green interval is recommended.

4.2 Adaptable Signal Timing Simulations

In this case, our signal control model represents a 2-phase independent intersection based on actuated control characterized by responsive operation. While the preceding model assigns the right of way according to previous records of traffic demands, the right of way in this model is based on current traffic conditions. As in the preceding model the green interval for each phase is predefined, in this model the green interval for each phase is defined on the basis of ongoing traffic conditions. In other words, the actuated control implemented in this model reproduces a signal control more adaptable to changing traffic volumes.

Figure 3 shows the scenario in which this model is portrayed. As we can see, Figures 2 and 3 are very similar. In both figures the sliders and controls are almost the same except that we have removed from Figure 2 the last two sliders to set up the green and yellow interval respectively. Instead of these two sliders, a new slider has been defined for the customization of the cycle length. In this way, this slider allows to set up the time in seconds to complete one full cycle of all signal indications.

The columns labeled as Green-East and Green-North in Table 2 represent the distribution of the green interval for each approach. Said in another way, these

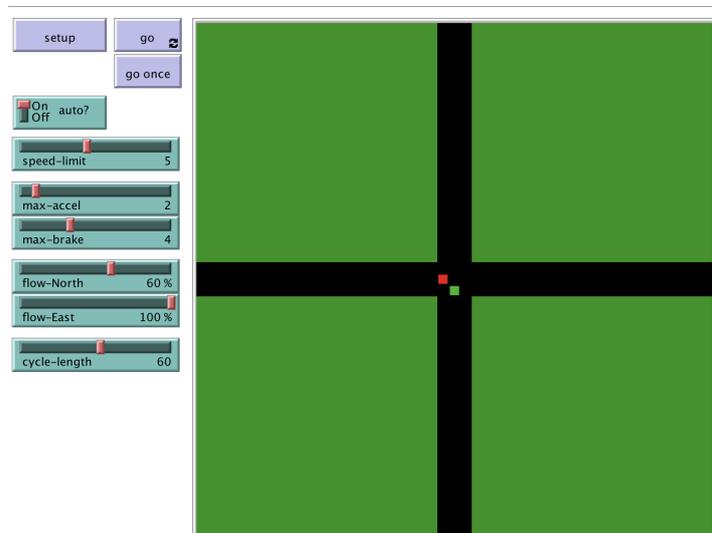


Fig. 3. Intersection-cycle-2-phases.

columns illustrate how the green interval for each phase is based on current traffic conditions. The values of these columns have been obtained from the equation 3 to calculate the green interval in terms of the cycle-length. For example, for a cycle-length of 32, when the traffic distribution in the intersection is 60% on North and 100% on East, the green values are 12 and 20 respectively. Thus, we have in this model a more efficient distribution of the green interval for each phase: at higher traffic volume, higher green interval.

However, when the traffic volume is increased to 80% on the North approach, the representation of the intersection as junction of a minor street and a major street is not so obvious: the difference in traffic volume between the streets has now decreased. So in this new traffic distribution, and under the same cycle-length, the green intervals obtained are 14 and 18 respectively. Even though a higher green interval has been allotted to the approach with higher volume, the green interval for the major street has been decreased. And lastly, when the traffic volume is increased to 100% on the North approach, we are simulating an intersection of streets with equal traffic distribution. In this case, the green intervals obtained are 16 and 16 respectively. Since the traffic volume on each approach is identical, the green interval assigned for each approach is exactly the same.

Table 2 also shows the number of stopped cars for each traffic distribution. From one perspective, the number of stopped cars are similar to the values shown in Table 1: as long as the traffic volume on the North approach (values of 60 and 80) is less than the traffic volume on the East approach, the number of stopped cars on the major street (East road) is higher than the number of stopped cars on the minor street (North road). However, an important difference is worth to

Table 2. Cycle-length 32.

		Flow of cars		Stopped cars				Green	
Traffic Distribution	Ticks	East	North	East	North	%East	%North	East	North
60 North - 100 East	600 (15)	272	162	415	186	1.53	1.15	20	12
	1200 (30)	532	319	807	298	1.52	0.93		
	2400 (60)	1046	627	1647	530	1.57	0.85		
80 North - 100 East	600 (15)	248	196	414	312	1.67	1.59	18	14
	1200 (30)	480	383	833	565	1.74	1.48		
	2400 (60)	941	752	1674	894	1.78	1.19		
100 North - 100 East	600 (15)	223	231	419	413	1.88	1.79	16	16
	1200 (30)	423	443	872	849	2.06	1.92		
	2400 (60)	848	863	1718	1695	2.03	1.96		

be noticed: the green interval assigned for the traffic volume on both streets is not enough. And of course, things getting worst when the traffic volume on both streets is equal. Platoons of slow-moving vehicles are formed around both streets and also disturbed by a delay. Thus, a substantial increase on the cycle length is recommended.

4.3 Discussion

In the previous section, the results of the simulations have shown the performance of two different signal control methods. The results were analyzed by comparing different traffic distributions for each signal control model. Now, in this section we examine the results by comparing the same traffic distributions under two different signal control methods. In other words, instead of analyzing the values within a particular table of simulations, we now inspect the values between the tables of different signal control results.

We look at Table 1 and Table 2 how, as long as the traffic volume on the North approach (values of 60 and 80) is less than the traffic volume on the East approach (value of 100), the number of stopped cars on the major street (East) is higher than the number of stopped cars on the minor street (North). However, even though this uniformity is observed in both signal control methods, two important differences must be noticed. First, the number of stopped cars on the East approach in Table 2 is less than the corresponding value in Table 1. Second, the number of stopped cars on the North approach in Table 2 is greater than the corresponding value in Table 1. What does this mean? On the one hand, the adaptable signal control decreases the delay on the East approach, but on the other hand the delay has been increased on the North approach.

Even though the problem of density still exists on the major road, the formation of platoons starts to arise on the minor road.

Now, when the traffic distribution on both streets is equal, the problem of density has been extended to both roads. By simulating a traffic intersection of streets with equal traffic volumes we clearly see how the number of stopped cars on the North approach in Table 1 is similar to the corresponding value in Table 2. And we can observe the same regularity on the East approach. Thus, a plausible option to avoid platoons of slow-moving vehicles around both streets is a substantial increase on the current cycle length.

To verify this assumption, Table 3 shows the results of the simulation for a cycle length of 48. Regardless of the traffic distribution, a substantial decrease in the number of stopped cars on both streets is observed when we compare the corresponding values of Table 2 and Table 3. For example, when the traffic volume in the intersection is represented as a junction of a minor and a major street the vehicles move without delay. Even when traffic volume on both streets is equal the vehicles do not experiment a disturbing delay. In this way, an increase on the cycle length from 32 to 48 has proved to be a good alternative for a less congested flow.

Table 3. Cycle-length 48.

		Flow of cars		Stopped cars				Green	
Traffic Distribution	Ticks	East	North	East	North	%East	%North	East	North
60 North - 100 East	840 (15)	385	231	407	185	1.06	0.80	30	18
	1680 (30)	753	449	814	303	1.08	0.67		
	3360 (60)	1501	898	1625	634	1.08	0.71		
80 North - 100 East	840 (15)	356	281	406	270	1.14	0.96	27	21
	1680 (30)	697	554	809	672	1.16	1.21		
	3360 (60)	1373	1089	1638	1476	1.19	1.36		
100 North - 100 East	840 (15)	315	319	416	407	1.32	1.28	24	24
	1200 (30)	615	626	830	821	1.35	1.31		
	3360 (60)	1228	1238	1662	1664	1.35	1.34		

According to the results of multiple simulations, we can clearly see what conditions are more appropriated for a particular signal control method. As Gartner claims each signal control method has its unique advantages and disadvantages [3]. Since our experimentation was carried out to analyze the implications of multiple traffic demands, we were able to verify why a fixed signal control is more appropriated when we have a predictable traffic demand. In our case, when the intersection is represented as a junction of a minor and a major street, that

is, and the traffic flow on the North approach is significantly less than the East approach, a fixed green interval of [16 - 18] for the North approach is enough. As for the major street, that is, the East approach, a fixed green interval of [28-30] is acceptable.

However, when the traffic distribution on both streets of the intersection is almost equal, an adaptable signal control is the best option. Since the traffic volume in a particular street might be higher than the other one in a specific moment, and it might also be lower a few moments later, the traffic current conditions are crucial for setting the time interval for each signal indication. In our case, when the traffic volume on both streets is similar, we tried two cycle-lengths: 32 and 48. As we can see in Table 3, an adaptable and better distribution of the green intervals has been obtained with a cycle length of 48. As we previously said, a cycle length of 48 has proved to be a good alternative for a less congested flow.

5 Conclusions and Future Work

This paper has investigated the basic elements to be considered for the control of a traffic light by using modelling based on agents. We focus our attention in how the flow of vehicles in each street of the junction influences the control of a traffic light. By trying different values of traffic flow while keeping the cycle-length fixed, the user can analyze the impact of the traffic flow in each direction under a particular cycle-length at the traffic signal. In this way, the traffic engineer is able to verify hypothesis as to how a particular configuration of the pertinent parameters works.

There are two directions in which our model could be improved: to increase the number of phases in the traffic junction and the coordinated operation of two or more signalized intersections. To extend the model to not only increase the number of phases but also include left turning movements sounds rather interesting. Likewise, to extend the model to consider parameters such as the distance between two or more signals and the required offsets to coordinate the operation of adjacent intersections on the same cycle length sounds also appealing. To work in these directions will contribute to the robustness of our signal timing model.

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